Huygens Subgridding for Frequency-Dependent Finite-Difference Time-Domain Method

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Outline

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Motivation

- Character of electromagnetic wave propagation
- Modern engineering projects:
  - very large problem size
  - fine geometric details
  - dielectric material properties.
- Answer: efficient large-scale Maxwell's equations solver

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Finite-Difference Time-Domain Method

Regular FDTD [1]:

$$\frac{\partial \boldsymbol{H}}{\partial t} = -\frac{1}{\mu}\nabla \times \boldsymbol{E}, \quad \frac{\partial \boldsymbol{D}}{\partial t} = \nabla \times \boldsymbol{H}$$

Debye relaxation:

$$\boldsymbol{D} = \varepsilon_0 \varepsilon_r \boldsymbol{E} = \varepsilon_0 \left(\varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + \jmath\omega\tau_1} + \frac{\sigma}{\jmath\omega\varepsilon_0} \right) \boldsymbol{E}$$

Courant-Friedrichs-Lewy stability [2]:

$$S \equiv \Delta t c \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}, \quad S \leq 1$$

DEH Approach

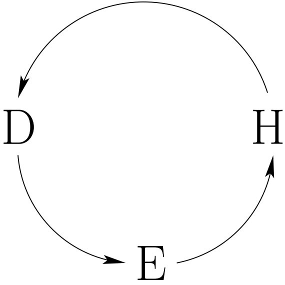
calc $H_*^{n-\frac{1}{2}}$

calc D_*^n

calc D_{src}^n

calc E_*^n

calc E_{ABC}^n



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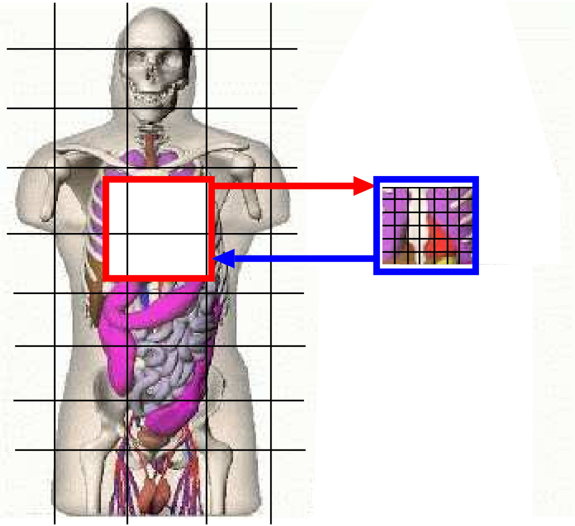
Subgridding Example



General Subgridding

- FDTD efficiency improvement $\longrightarrow$ *Subgridding*
- Idea: simulation space $= n \times$ subspaces with different Δt and Δs
- Key issues:
  - compromise: efficiency $\longleftrightarrow$ accuracy
  - subgridding interface: stability and accuracy
  - parallelisation
  - number of grid refinement levels
  - subgridding ratio $r = \frac{\Delta s_a}{\Delta s_b}$, $(r_{sim} \leq 7)$
  - temporal increments:

$$\begin{aligned} &\text{synchro unistep} && \longrightarrow \Delta t_a = \Delta t_b \\ &\text{synchro multistep} && \longrightarrow \Delta t_a = r \cdot \Delta t_b. \end{aligned}$$

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Huygens Subgridding

- Interface: Huygens Surfaces (HS), little reflection
- Influence: equivalent currents
- Time step: synchro multistep $\longrightarrow \Delta t_a = r \cdot \Delta t_b$
- Subgridding ratio: $r = \frac{\Delta s_a}{\Delta s_b} = \frac{\Delta t_a}{\Delta t_b}$, $(r_{sim} \leq 50)$.

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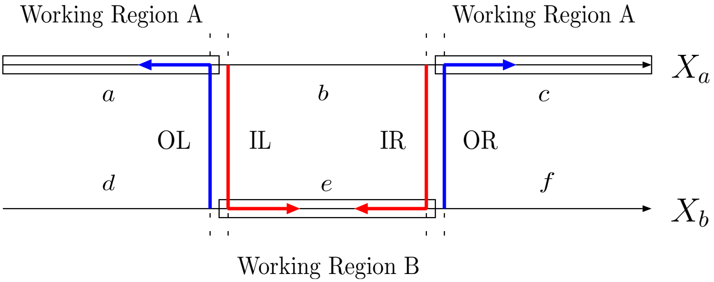
HSG Inner and Outer Surfaces

Single space:

Decomposition: non-working regions $\longrightarrow$ no influence



HSG Decomposition into Subspaces

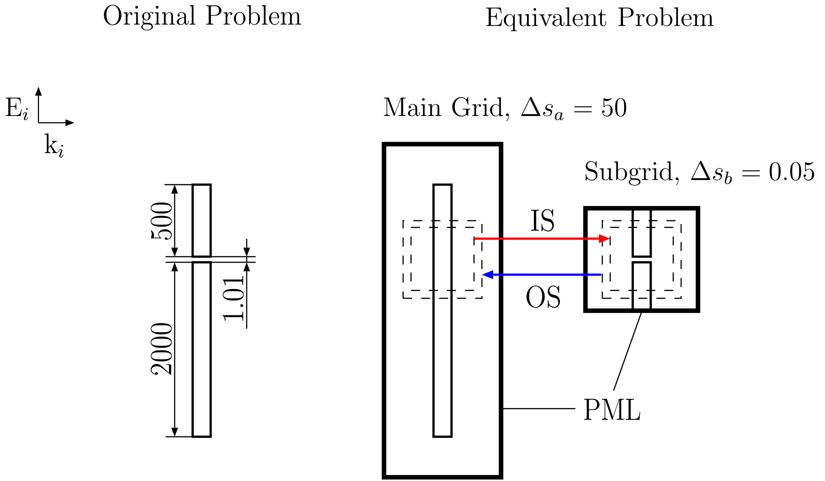


Figure: Thin Slot, 2D, $r = 99$ [3]

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Input Parameters

Parameter	HSG	DEH (b)
nx	88	264
t_{max}	1000	3000
r	3	–
alSIS	26	–
source type	soft	soft
f_w	3.1 GHz	3.1 GHz
S	0.93	0.93
Tx	19	57

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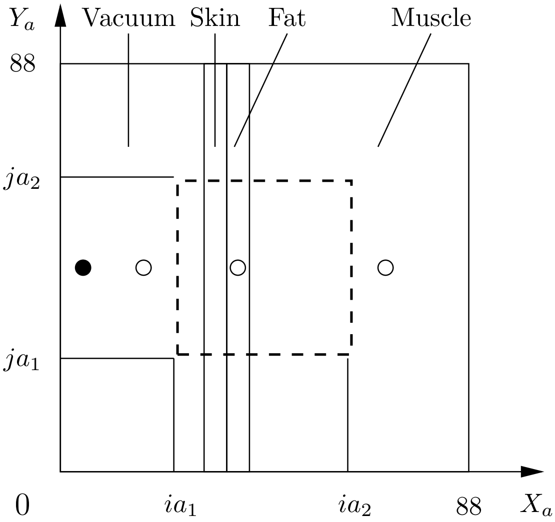
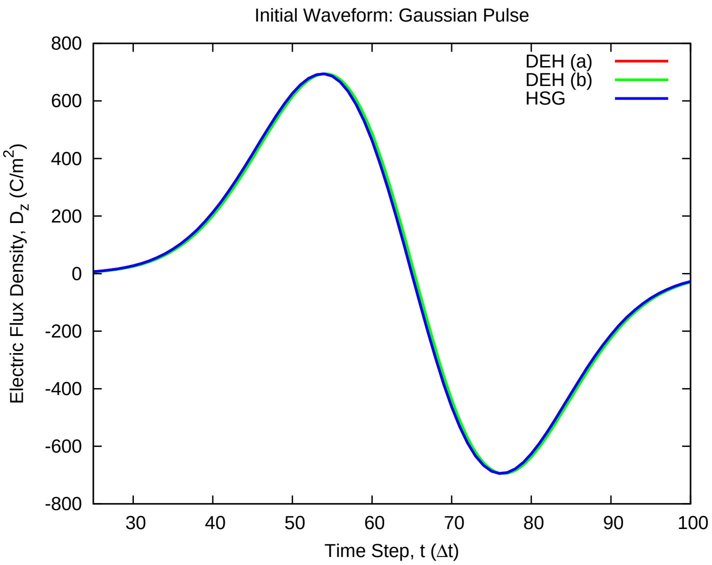
Debye Media Parameters

Medium	$\sigma, \left[\frac{S}{m}\right]$	$\varepsilon_s,$	$\varepsilon_\infty,$	$\tau_1, [s]$
vacuum	0	1	1	0
fat	0.03434	6.12803	3.7310	$1.7996 \cdot 10^{-11}$
skin	0.41326	76.5550	19.534	$1.6742 \cdot 10^{-11}$
muscle	0.69874	67.2970	21.015	$1.3560 \cdot 10^{-11}$

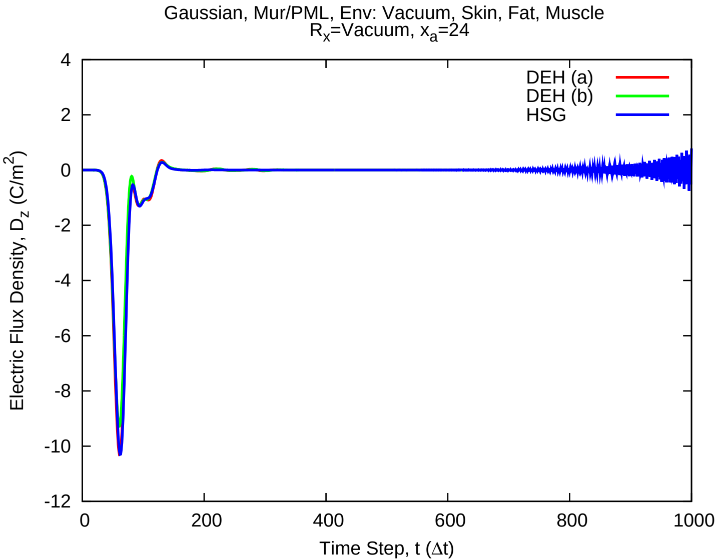
Scenario Setting, 2D



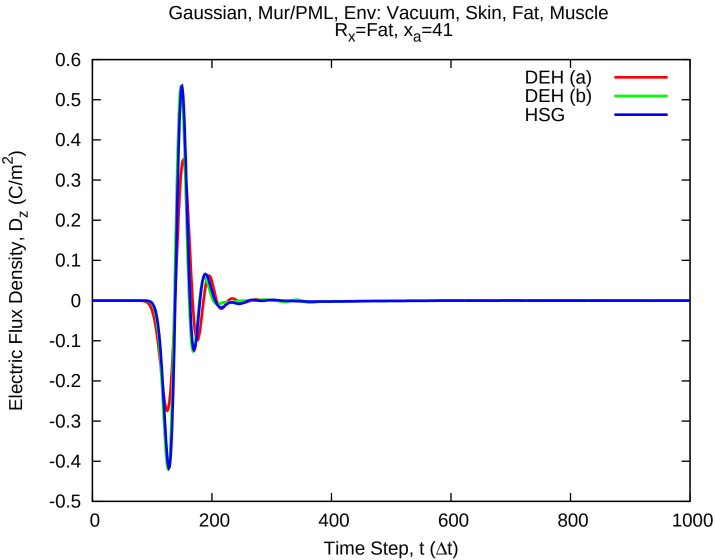
Initial Waveform: Gaussian Pulse



$R_x = Vacuum, x_a = 24$

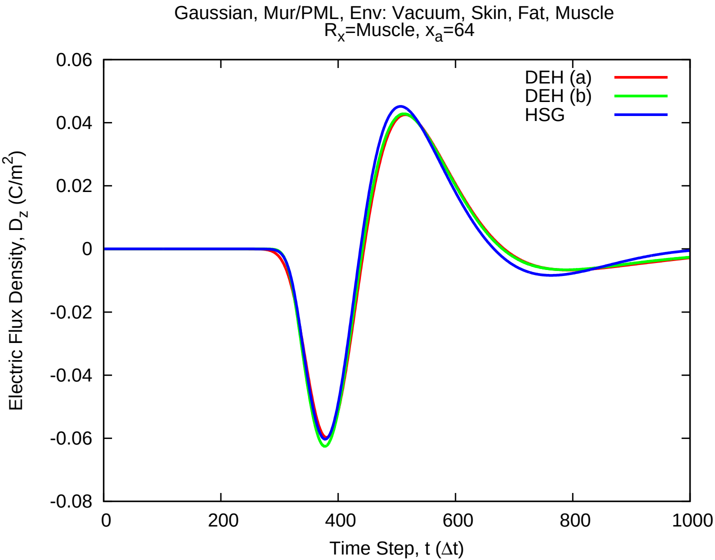


$R_x = Fat, x_a = 41$



Gaussian, Mur/PML, Env: Vacuum, Skin, Fat, Muscle
Rx=Fat, xa=41
DEH (a)
DEH (b)
HSG
Electric Flux Density, Dz (C/m2)
Time Step, t (Δt)
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
200
400
600
800
1000

$R_x = Muscle, x_a = 64$



Instability in HSG

Dispersion relation:

$$\sin\left(\frac{\omega\Delta t}{2}\right) = \frac{c\Delta t}{\Delta x}\sin\left(\frac{k_x\Delta x}{2}\right).$$

Wave frequency:

$$f_w = \begin{cases} \text{low}, & k_x \in \mathbb{R}, \text{ travelling waves}, \\ \text{high}, \; \sin\left(\frac{k_x\Delta x}{2}\right) > 1, & k_x \in \mathbb{C}, \text{ evanescent waves}. \end{cases}$$

Instability frequency [3]:

$$f_{inst} \approx f_{tran} = \frac{1}{\pi\Delta t}\arcsin\left(\frac{c\Delta t}{\Delta x}\right).$$

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Filtering in HSG

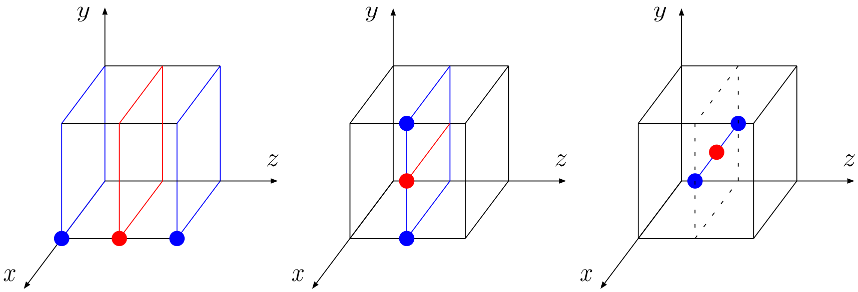
Filter equivalent currents IS:

$$\overline{E}_{a(IS)} = \frac{E_{a(IS-1)} + E_{a(IS)}}{2} = \frac{1}{2}E_{a(IS-1)} + \frac{1}{2}E_{a(IS)}.$$

Cosine filter:

$$F_1(k_x) = \frac{e^{\jmath k_x \Delta x_a} + 1}{2} = e^{\frac{\jmath k_x \Delta x_a}{2}} \cos\left(\frac{k_x \Delta x_a}{2}\right).$$

Filtering Implementation



Conclusion

- HSG successfully applied to FD–FDTD 3D:
  - efficient Maxwell's equations solver
  - dielectric material properties
  - late-time instable.

Future Work:

- apply method to human body model $\longrightarrow$ defibrillation parameters
- experiment with different digital filters
- parallelise HSG–FD–FDTD 3D.

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Discussion:

- thank you for attention
- questions and answers.