Chapter 2

Mathematics in LaTeX

2.1 Including Mathematics

- Text, in-line mathematics with `$...$`, `$a^2 + b^2 = c^2$`, $a^2 + b^2 = c^2$
- Displayed, centred mathematics with `\[...\]`, `\[a^2 + b^2 = c^2\]`,

$$a^2 + b^2 = c^2$$

- Advanced mathematics environments with `amsmath` or `mathtools` packages

```
\usepackage{amsmath}
\usepackage{mathtools}
```

- AMS symbols and theorems with `amssymb` and `amsthm` packages

```
\usepackage{amssymb}
\usepackage{amsthm}
```

2.2 Mathematics Commands

- Greek letters
  - `A, \alpha, B, \beta, \Gamma, \gamma, \Pi, \pi, \Sigma, \sigma`
  - A, α, B, β, Γ, γ, Π, π, Σ, σ
  - `u = A \sin (\omega t + \gamma)`, $u = A\sin(\omega t + \gamma)$
- Keyboard symbols `+ - = ! / ( ) [ ] < > ':`
- Command symbols
  - `\forall \in \exists \leq \approx \infty`
  - $\forall$ $\in$ $\exists$ $\leq$ $\approx$ ∞
- Superscript (power) `^`, `a^n`, a^n
- Subscript (index) `_`, `a_n`, a_n
- Operators (functions)
  - `\sin, \cos, \tan, \cot, \log, \exp, \lim`
  - `\sin^2 \alpha + \cos^2 \alpha = 1`
  - $\sin^2 \alpha + \cos^2 \alpha = 1$
- Fractions `\frac{1}{256}`, $\frac{1}{256}$
- Simple fractions `1/512`, $1/512$

- Powers and indices `^1/_{1024}`, $^1/_{1024}$
- Roots `\sqrt[]{}`, `\sqrt[n]{a}`, $\sqrt[n]{a}$
- Sums `\sum`, `\sum_{n=1}^{256} a_n`, $\sum_{n=1}^{256} a_n$
- Integrals `\int`, `\int_a^b f(x) \, {\mathrm d} x`, $\int_a^b f(x) \, {\mathrm d} x$
- Partial differential `\partial`, `\frac{\partial u}{\partial x}`, $\frac{\partial u}{\partial x}$
- Brackets
  - `( [ \{ | \| \langle \lfloor \lceil`
  - $(\quad [\quad \{ \quad | \quad \| \quad \langle \quad \lfloor \quad \lceil$
- Automatic sizing of brackets with the `\left ( ... \right )` commands
  - `\left( \frac{x}{a} \right)`, $\left(\frac{x}{a} \right)$
- Matrix environments `matrix`, `pmatrix`, `bmatrix`, `Bmatrix`, `vmatrix`, `Vmatrix`

```
\[
  \begin{matrix}
    a_{1,1} & a_{1,2} & a_{1,3} \\
    a_{2,1} & a_{2,2} & a_{2,3} \\
    a_{3,1} & a_{3,2} & a_{3,3}
  \end{matrix}
\]
```

$$\begin{matrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{matrix}$$

  - Column separator &
  - Row separator \\
- Text in math mode with the `\text{}` command
  - `1~\text{byte} = 8~\text{bits}`, $1~\text{byte} = 8~\text{bits}$
- Accents in math mode
  - `\hat{a}`, `\bar{b}`, `\dot{c}`, `\ddot{d}`, `\vec{e}`
  - $\hat{a}$, $\bar{b}$, $\dot{c}$, $\ddot{d}$, $\vec{e}$
- Horizontal spacing
  - `\, \: \; \quad`
  - `a \, b` `a \: b` `a \; b` `a \quad b`
  - $a \, b \quad a \: b \quad a \; b \quad a \quad b$
- Dots
  - horizontal `\cdots \ldots \dots`,[1] $\cdots \quad \ldots \quad \dots$
  - vertical `\vdots`, $\vdots$
  - diagonal `\ddots`, $\ddots$

[1] Command `\dots` is only available with the `amsmath` package.

2.3 Defining Theorems

- Theorems package `amsthm`
  - Define a theorem in document preamble with `\newtheorem{theorem}{Theorem}`
  - Use `theorem` environment in the main text

```
\begin{theorem}
  In any right triangle, the area of the square whose side
  is the hypotenuse...
\end{theorem}
```

Theorem 1. *In any right triangle, the area of the square whose side is the hypotenuse...*

  - Use `proof` environment for the proof

```
\begin{proof}[Pythagorean theorem]
  ...
\end{proof}
```

Pythagorean theorem. ... □

  - Define custom environments with the `\newtheorem{}{}` command

```
\newtheorem{lemma}{Lemma}
```

2.4 Equation Environments

- Normal equations with `equation, equation*` environments

```
\begin{equation}
  a_n = a_1 + (n-1)r \quad \text{and} \quad s_n = \frac{n(a_1 + a_n)}{2}
  \label{E:arithmeticProgression}
\end{equation}
```

$$a_n = a_1 + (n-1)r \quad \text{and} \quad s_n = \frac{n(a_1 + a_n)}{2} \tag{2.1}$$

- Multiple aligned equations with `align, align*` environments

```
\begin{align}
  \label{E:MaxwellEqs}
  \frac{\partial B}{\partial t} &= - \nabla \times E - M \\[12pt]
  \frac{\partial D}{\partial t} &= \phantom{-} \nabla \times H - J \notag
\end{align}
```

$$\begin{aligned} \frac{\partial B}{\partial t} &= - \nabla \times E - M \qquad (2.2) \\[12pt] \frac{\partial D}{\partial t} &= \nabla \times H - J \end{aligned}$$

- Column alignment with `&`, line break with `\\` or `\\[12pt]`

- Suppression of equation numbers with `\notag`
- Long equations spanning multiple lines with `multline`, `multline*` environments

```
\begin{multline}
  f(x) = f(a) + \frac{x-a}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a)
               + \frac{(x-a)^3}{3!} f^{(3)}(a) + \frac{(x-a)^4}{4!} f^{(4)}(a)
               + \frac{(x-a)^5}{5!} f^{(5)}(a) + \\[12pt]
               + \frac{(x-a)^6}{6!} f^{(6)}(a) + \frac{(x-a)^7}{7!} f^{(7)}(a)
               + \dots + \frac{(x-a)^n}{n!} f^{(n)}(a) + \dots
  \label{E:TaylorSeries}
\end{multline}
```

$$f(x) = f(a) + \frac{x-a}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f^{(3)}(a) + \frac{(x-a)^4}{4!} f^{(4)}(a) + \frac{(x-a)^5}{5!} f^{(5)}(a) + \\ + \frac{(x-a)^6}{6!} f^{(6)}(a) + \frac{(x-a)^7}{7!} f^{(7)}(a) + \dots + \frac{(x-a)^n}{n!} f^{(n)}(a) + \dots \quad (2.3)$$

- Control the line break with `\\` or `\\[12pt]`
- Comment equations with %
- Label equations with `\label{}` command
- Refer to equations with `\eqref{}` command

2.5 User-Defined Commands

- Create custom commands with `\newcommand{name}{command}`
- Commands must be defined in the document preamble
  - No arguments:
    `\newcommand{\light}{c_0}` `\light` c_0
  - One argument:
    `\newcommand{\veci}[1]{u_{#1}}` `\veci{n}` u_n
  - Two arguments:
    `\newcommand{\mati}[2]{a_{#1, #2}}` `\mati{i}{j}` $a_{i,j}$

2.6 Exercises

1. Start a new article and activate AMS packages `amsmath`, `amssymb`, `amsthm`.
2. Write Pythagorean theorem using in-line math mode `$...$` and command `\sqrt{}`:

$$c = \sqrt{a^2 + b^2}$$

3. Write the freshman's dream equation using in-line math mode and not equals sign:

$$(x + y)^n \neq x^n + y^n$$

4. Write laws of sines, cosines and tangents using displayed math mode `\[...\]`:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$$

$$a^2 = b^2 + c^2 - 2\,bc\cos\alpha$$

$$\frac{a+b}{a-b} = \frac{\tan\frac{1}{2}(\alpha+\beta)}{\tan\frac{1}{2}(\alpha-\beta)}$$

5. Write formulae of the geometric progression using the `equation` environment:

$$a_n = a_1 q^{n-1} \quad \text{and} \quad s_n = \frac{a_1(q^n - 1)}{q-1} \tag{2.4}$$

6. Give example of a sum of a geometric progression using the capital sigma notation:

$$1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots = \sum_{n=1}^{N} \frac{1}{2^n} = \frac{1}{1-\frac{1}{2}} = 2 \tag{2.5}$$

7. Write the sophomore's dream identities discovered by Johann Bernoulli in 1697 using the `align` environment. State approximate values of the identities up to the 10th digit:

$$\int\limits_0^1 x^{-x}\,\mathrm{d}x = \sum_{n=1}^{\infty} n^{-n} \approx 1.2912859970 \tag{2.6}$$

$$\int\limits_0^1 x^{x}\,\mathrm{d}x = \sum_{n=1}^{\infty} (-1)^{n+1} n^{-n} = -\sum_{n=1}^{\infty} (-n)^{-n} \approx 0.7834305107 \tag{2.7}$$

8. Write the definition of Maclaurin series using the `equation` environment and the capital sigma notation:

$$f(x) = f(0) + \sum_{n=1}^{N} \frac{x^n}{n!} f^{(n)}(0) \tag{2.8}$$

9. Write the definition of Maclaurin series using the `multline` environment. Create a custom command to write a single term of the series and show the first seven terms:

$$\begin{multline} f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f^{(3)}(0) + \frac{x^4}{4!}f^{(4)}(0) + \frac{x^5}{5!}f^{(5)}(0) + \\ + \frac{x^6}{6!}f^{(6)}(0) + \frac{x^7}{7!}f^{(7)}(0) + \dots + \frac{x^n}{n!}f^{(n)}(0) + \dots \end{multline} \tag{2.9}$$

10. Write a three by four matrix, apply the `pmatrix` or `bmatrix` environments for round or square brackets:

$$M = \begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} \end{pmatrix} \tag{2.10}$$

11. Give definition of the gradient in Cartesian coordinate system, use commands `\nabla` and `\partial`:

$$\nabla U = \frac{\partial U}{\partial x} i + \frac{\partial U}{\partial y} j + \frac{\partial U}{\partial z} k \tag{2.11}$$

2.7 Exercises for Independent Work on Advanced Mathematical Notation

1. **Limit of a multivariable function.** Use `amsmath` package, limit operator `\lim` and command `\substack{}`. Apply the linebreak command `\\` inside of the `\substack{}` in order to switch to the next line. Right directed arrow is typeset with a `\rightarrow` command.

$$A = \lim_{\substack{x\rightarrow a\\ y\rightarrow b}} f(x,y). \tag{2.12}$$

2. **Derivative of a single variable function.** Use command `\Delta` to typeset a capital Greek letter delta.

$$f'(x) = \lim_{\Delta x\rightarrow 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}. \tag{2.13}$$

3. **Number *e*.** Use command `\infty` to produce an infinity symbol. Automatic sizing of brackets is possible with a pair of commands `\left(` and `\right)`. Italicised text in math-mode is accessible via a `\textit{}` command.

$$\lim_{x\rightarrow\infty}\left(1+\frac{1}{x}\right)^x = e = 2.718282\ldots \quad \textit{(irrational number).} \tag{2.14}$$

4. **Euler's constant *C*.** Use operator `\ln{}` to typeset a natural logarithm.

$$\lim_{n\rightarrow\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\ln n\right) = C = 0.577216\ldots \quad \textit{(Euler's constant).} \tag{2.15}$$

5. **Duality map $F(x_0)$.** Use commands `\{`, `\|`, `\la` and `\ra` as well as the manual sizing command `\Big` to typeset various brackets. A star might be produced with a `\star` command [5].

$$F(x_0) = \Big\{ f_0 \in E^\star; \quad \|f_0\| = \|x_0\| \text{ and } \langle f_0, x_0\rangle = \|x_0\|^2 \Big\}. \tag{2.16}$$

6. **Gamma function.** Use command `\Gamma` to write a capital Greek letter gamma. Integral symbol is produced with the `\int` command. Apply `cases` environment of the `amsmath` package to create a definition of a piecewise function. Syntax inside of the `cases` environment is the same as in the `tabular` environment (symbols `&` and `\\`). Text in math-mode is written inside of a `\text{}` command.

$$\Gamma(x) \begin{cases} = \int\limits_0^\infty e^{-t} t^{x-1}\,\mathrm{d}t \quad \textit{(Euler's integral)} & \text{only if} \quad x>0, \\ = \lim\limits_{n\rightarrow\infty} \dfrac{n!\,n^{x-1}}{x(x+1)(x+2)\ldots(x+n-1)} & \text{for any} \quad x. \end{cases} \tag{2.17}$$

7. **Newton's binom.** Use `multline` environment of the `amsmath` package and the linebreak command `\\` to split the formula at the appropriate place.

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \dots + \\ + \frac{n(n-1)\dots(n-m+1)}{m!}a^{n-m}b^m + \dots + nab^{n-1} + b^n. \quad (2.18)$$

8. **Bernoulli's numbers.** Plus/minus and minus/plus symbols are produced with the `\pm` and `\mp` commands in math-mode. Greek letter pi is written with a `\pi` command.

$$1 - \frac{1}{2^{2k}} + \frac{1}{3^{2k}} - \frac{1}{4^{2k}} + \dots \pm \frac{1}{n^{2k}} \mp \dots = \frac{\pi^{2k}(2^{2k-1}-1)}{(2k)!}B_k. \quad (2.19)$$

9. **Euler's numbers**

$$1 - \frac{1}{3^{2k+1}} + \frac{1}{5^{2k+1}} - \frac{1}{7^{2k+1}} + \dots \pm \frac{1}{(2n-1)^{2k+1}} \mp \dots = \frac{\pi^{2k+1}}{2^{2k+2}(2k)!}E_k. \quad (2.20)$$

10. **Stokes' formula.** Define two custom commands: one without arguments to typeset a differential over a given variable $(\mathrm{d}x)$ and another with two arguments to typeset a fraction of partial differentials $\left(\frac{\partial Q}{\partial x}\right)$. Hamiltonian (nabla operator) is written with a `\nabla` command and a cross product—with a `\times` command. Closed contour integral is given with an `\oint` command of the `amsmath` package and bold font face in math-mode is set with a `\mathbf{}` command. Use the `align` environment to write Stokes' formula in Cartesian coordinates and in vector form.

$$\int\limits_K P\,\mathrm{d}x + Q\,\mathrm{d}y + R\,\mathrm{d}z = \int\limits_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathrm{d}x\,\mathrm{d}y + \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\mathrm{d}y\,\mathrm{d}z + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\mathrm{d}z\,\mathrm{d}x, \quad (2.21)$$

$$\int\limits_\Sigma \nabla \times \mathbf{V}\,\mathrm{d}S = \oint\limits_C \mathbf{V}\,\mathrm{d}\mathbf{r}. \quad (2.22)$$

11. **Curl theorem applied to Maxwell's equations.** Triple integral is written with `\iiint` command and double closed surface integral with—`\oiint`. Matrices in square brackes are typeset with `bmatrix` environment, while commands `\hat{}` and `\vec{}` produce the accents required.

$$\iiint\limits_V \nabla \times \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} \mathrm{d}V = \oiint\limits_S \begin{bmatrix} \hat{n} \times \vec{H} \\ \hat{n} \times \vec{E} \end{bmatrix} \mathrm{d}S. \quad (2.23)$$

12. **Green's formula**

$$\int\limits_K P\,\mathrm{d}x + Q\,\mathrm{d}y = \iint\limits_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathrm{d}x\,\mathrm{d}y. \quad (2.24)$$

13. **Green's theorems**

$$\int\limits_{\Sigma} U_1 \nabla U_2 \, \mathrm{d}S = \int\limits_{v} (U_1 \Delta U_2 + \nabla U_1 \nabla U_2) \, \mathrm{d}v, \tag{2.25}$$

$$\int\limits_{\Sigma} (U_1 \nabla U_2 - U_2 \nabla U_1) \, \mathrm{d}S = \int\limits_{v} (U_1 \Delta U_2 - U_2 \Delta U_1) \, \mathrm{d}v, \tag{2.26}$$

$$\int\limits_{\Sigma} \nabla U \, \mathrm{d}S = \int\limits_{v} \Delta U \, \mathrm{d}v \quad (\textit{for} \quad U_1 = 1). \tag{2.27}$$

14. **Ostrogradsky–Gauss' formula**

$$\iiint\limits_{V} \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) \mathrm{d}V = \iint\limits_{S} P \, \mathrm{d}y \, \mathrm{d}z + Q \, \mathrm{d}z \, \mathrm{d}x + R \, \mathrm{d}x \, \mathrm{d}y, \tag{2.28}$$

$$\int\limits_{v} \nabla \mathbf{V} \, \mathrm{d}v = \oint\limits_{\Sigma} \mathbf{V} \, \mathrm{d}S. \tag{2.29}$$

15. **Fresnel integrals.** Apply a custom command without arguments to typeset a differential over a given variable $\mathrm{d}x$. Use commands `\left(` and `\right)` to create a pair of round brackets inside of the sine and cosine operators. Equivalents symbol $\equiv$ is accessible via an `\equiv` command.

$$C(u) + i S(u) = \int\limits_0^u e^{i\pi x^2/2} \, \mathrm{d}x = \int\limits_0^u \cos\left(\frac{1}{2}\pi x^2\right) \mathrm{d}x + i \int\limits_0^u \sin\left(\frac{1}{2}\pi x^2\right) \mathrm{d}x, \tag{2.30}$$

where

$$C(u) \equiv \int\limits_0^u \cos\left(\frac{1}{2}\pi x^2\right) \mathrm{d}x, \quad S(u) \equiv \int\limits_0^u \sin\left(\frac{1}{2}\pi x^2\right) \mathrm{d}x. \tag{2.31}$$

16. **Signum function ★** Use `amsmath` package and `\DeclareMathOperator{\sgn}{sgn}` command to define a signum operator `sgn`. Command `\phantom{}` is necessary to insert "phantom" spaces in front of numbers 0 and 1.

$$y = \operatorname{sgn} x, \quad y = \begin{cases} -1 & \text{if} \quad x < 0, \\ 0 & \text{if} \quad x = 0, \\ 1 & \text{if} \quad x > 0. \end{cases} \tag{2.32}$$

17. **Independence of multivariable functions** ★ $u_1 = f_1(x_1, x_2, \ldots, x_n), \ldots, u_n = f_n(x_1, x_2, \ldots, x_n)$ Use `vmatrix` environment of the `amsmath` package to create a matrix with vertical bars. Partial differential is written with the `\partial` command. Equivalence and not equals symbols are produced with the commands `\equiv` and `\neq`. Define a custom command with two arguments to typeset a generic element of the matrix.

$$\begin{vmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} & \cdots & \dfrac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial f_n}{\partial x_1} & \dfrac{\partial f_n}{\partial x_2} & \cdots & \dfrac{\partial f_n}{\partial x_n} \end{vmatrix} \equiv \frac{D(f_1, f_2, \ldots, f_n)}{D(x_1, x_2, \ldots, x_n)} \neq 0. \tag{2.33}$$

18. **General expression of a third order derivative via differentials** ★ Use commands `\left[` and `\right]` to create a pair of square brackets, and environment `vmatrix*` from the package `mathtools` to typeset matrix expressions. Specify an optional column alignment parameter `[l]` of the `vmatrix*` environment as `\begin{vmatrix*}[l]`. Define a custom command with one argument `\dn{}` to typeset a differential of a given order. Apply a negative spacing command `\!` to compress horizontal space in between the symbols d^3 and x to produce $\mathrm{d}^3\!x$.

$$y''' = \left[\mathrm{d}x \begin{vmatrix} \mathrm{d}x & \mathrm{d}y \\ \mathrm{d}^3\!x & \mathrm{d}^3\!y \end{vmatrix} - 3\,\mathrm{d}^2\!x \begin{vmatrix} \mathrm{d}x & \mathrm{d}y \\ \mathrm{d}^2\!x & \mathrm{d}^2\!y \end{vmatrix} \right] : \mathrm{d}x^5. \tag{2.34}$$

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