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Huygens Subgridding with Filtering for Finite-Difference Time-Domain Method

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Introduction

- Building prototypes to verify electromagnetic (EM) characteristics of an object is expensive.
- Engineers use numerical methods to simulate EM wave propagation on computers.
- Finite-Difference Time-Domain (FDTD) is one of widely applied numerical schemes to approximate Maxwell's Equations.

Motivation

- Current electromagnetic problems are characterised by:
  - large problem size
  - fine geometry
  - variety of dielectric materials.

Research Objective and Focus

- Provide an efficient Maxwell's Equations solver capable of incorporating dielectric material properties and fine object geometry.
- This project focuses on:
  - Huygens Subgridding (HSG) application to increase computational efficiency of FDTD
  - Filtering to suppress late-time instabilities of HSG.

Huygens Subgridding Principles

Huygens Subgridding is a novel algorithm with the following unique properties [1]:

- Interface is implemented via Huygens Surfaces and produces little reflection.
- Influence from coarse to fine (a → b) and fine to coarse (b → a) grid is passed with equivalent currents.
- Temporal advancement scheme is the synchronised multistep: $\Delta t_a = r\Delta t_b$.
- Subgridding ratios for spatial and temporal domains are equal: $r = \frac{\Delta s_a}{\Delta s_b} = \frac{\Delta t_a}{\Delta t_b}$.

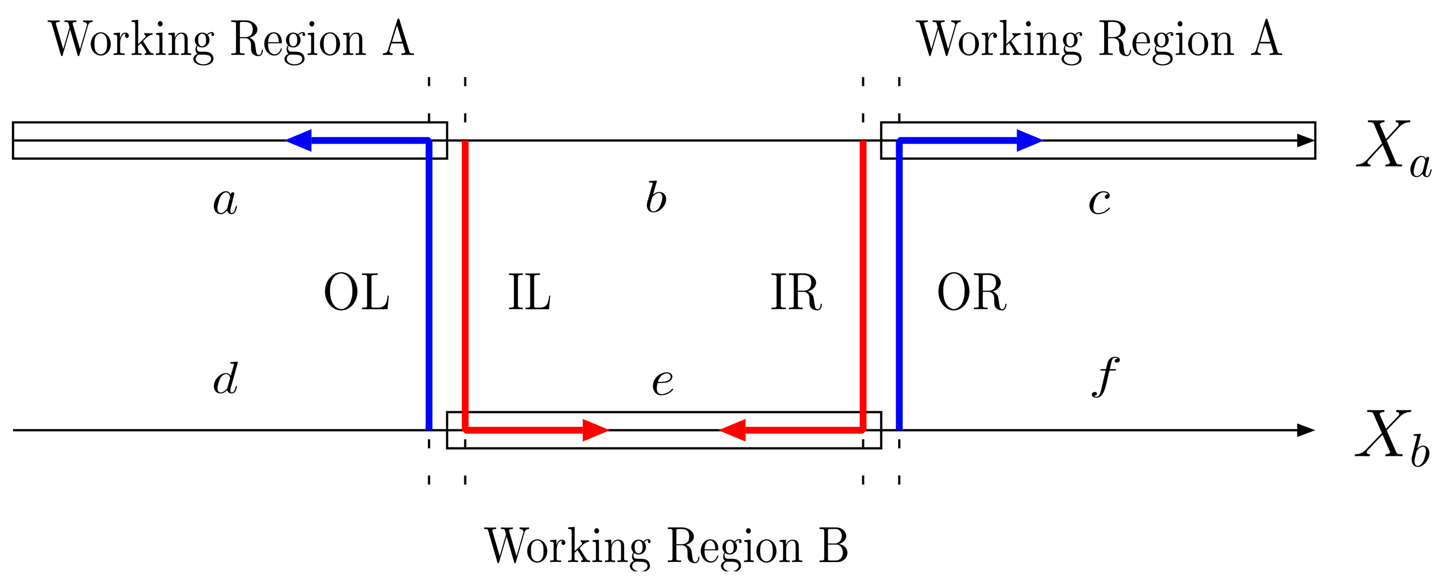

Figure 1: ***HSG Inner and Outer Surfaces.*** *HSG decomposes the simulation space into subspaces (X_a, X_b)—coarse and fine grids. Non-working regions (d, b, f) exert no influence on the simulation result. Equivalent currents pass electromagnetic energy from coarse to fine grid (Inner Huygens Surface IL, IR, red arrows) and fine to coarse grid (Outer Huygens Surface OL, OR, blue arrows).*

calc $H_{a(*)}$
calc $H_{a(interface)}$
calc $E_{b(*)}$
infl $IS : H_{a(IS)} \rightarrow E_{b(IS)}$
calc $G_{b(*)}$
for each time step $t_{b_1} \in [1, q]$ **do**
  calc $H_{b(*)}$
  infl $IS : E_{a(IS)} \rightarrow H_{b(IS)}$
  calc $E_{b(*)}$
  infl $IS : H_{a(IS)} \rightarrow E_{b(IS)}$
  calc $G_{b(*)}$
end for
infl $OS : E_{b(OS)} \rightarrow H_{a(OS)}$
calc $E_{a(*,src)}$
calc $E_{a(interface)}$
calc $G_{a(*)}$
calc $H_{b(*)}$
infl $IS : E_{a(IS)} \rightarrow H_{b(IS)}$
for each time step $t_{b_2} \in [1, q]$ **do**
  calc $E_{b(*)}$
  infl $IS : H_{a(IS)} \rightarrow E_{b(IS)}$
  calc $G_{b(*)}$
  calc $H_{b(*)}$
  infl $IS : E_{a(IS)} \rightarrow H_{b(IS)}$
end for
infl $OS : H_{b(OS)} \rightarrow E_{a(OS)}$

Algorithm 1: ***HSG Pseudocode.*** *Algorithm consists of two symmetric parts. Electric and magnetic fields are denoted as E, H and polarisation currents as G. Subscript in parentheses specifies the field location: "*" stands for the entire space. Inner and Outer Surface influences are denoted as IS, OS and marked with colours.*

Filtering

- All HSG simulations suffer from instabilities [2].
- Frequency of instability equals to transition frequency of travelling to evanescent waves:

$$f_{tran} = \frac{1}{\pi\Delta t} \arcsin\left(\frac{c\Delta t}{\Delta x}\right). \quad (1)$$

- Filters absorb instable frequencies f_{inst} and preserve the frequencies of interest f.
- Filters work as averaging functions and are applied to equivalent currents in IS:

$$\overline{E}_{a(IS)} = \frac{E_{a(IS-1)} + E_{a(IS)}}{2} = \frac{1}{2}E_{a(IS-1)} + \frac{1}{2}E_{a(IS)}. \quad (2)$$

- Mathematical equivalent to averaging is a cosine filter:

$$F_1(k_x) = \frac{\exp(jk_x\Delta x_a) + 1}{2} = \exp\left(\frac{jk_x\Delta x_a}{2}\right)\cos\left(\frac{k_x\Delta x_a}{2}\right). \quad (3)$$

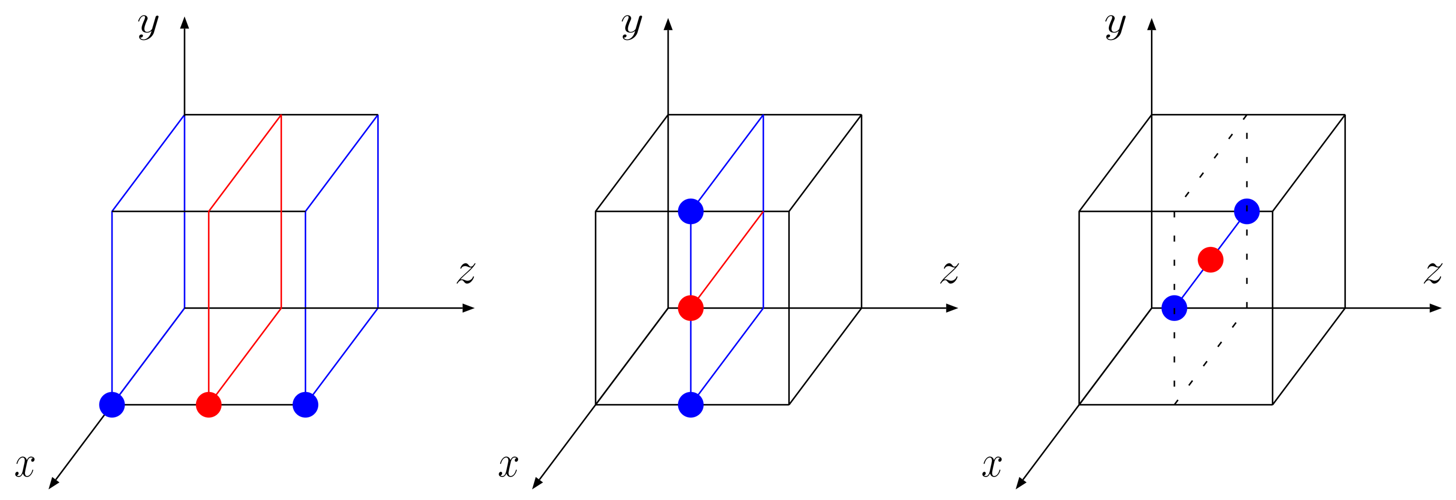

Figure 2: ***Single Point Filtering.*** *This Figure shows an interpolation process with Filter 33. Target point is located in the middle of the cube. Interpolation consists of three phases (from left to right): (i) volume to plane, (ii) plane to line and (iii) line to point reduction. Participating geometric elements are marked with colours, where the target object is red.*

Filter Performance

- One- and three-dimensional (1D, 3D) filters of orders 1, 3, 5 were implemented.
- All filters were tested on two verification scenarios:
  - vacuum only (100x)
  - vacuum, skin, fat, muscle (200x).

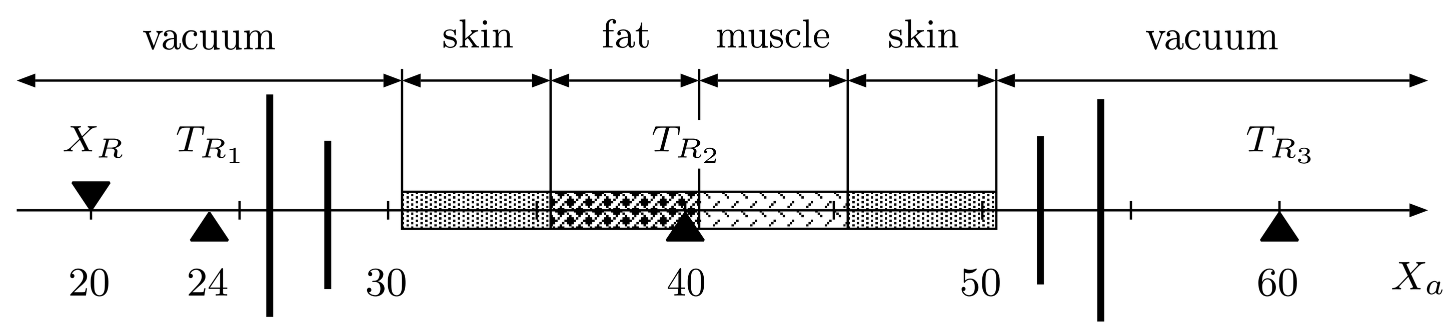

Figure 3: ***Scenario no. 200x, Setting.*** *Only 1D perspective of the scenario setting is shown. Four different materials were used in this setting: vacuum, skin, fat, muscle. Symbols X_R and T_R stand for the excitation source and observation locations.*

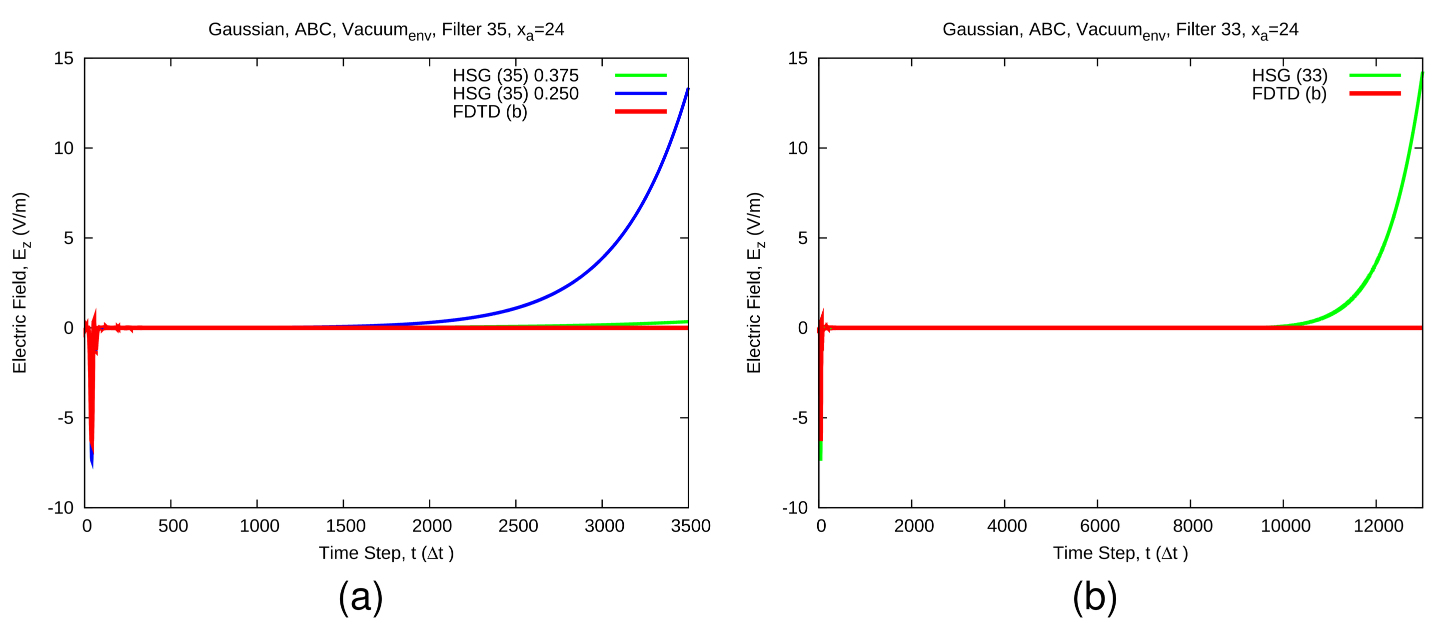

Figure 4:
(a) 100x, Filter 35, $x_a = 24$. *Interpolation coefficients influence filters' performance. Heavier central point improves 3D filters' suppression characteristics.*
(b) 100x, Filter 33, $x_a = 24$. *Filter 33 exhibits the best instability suppression qualities: $t_{inst,worst} = 5200$, $t_{inst,best} = 14000$ time steps.*

Verification Results

- Materials absorb part of the signal and delay the instability occurrence.
- HSG with $r = 3$ executes 8 times faster than all fine grid FDTD (b).
- Since filtering is done at every time step, it increases the computation time by 4.2% (Filter 15) and 20.5% (Filter 35).
- HSG uses approximately 5.5 times less memory than FDTD (b).

Summary

- HSG was successfully applied to FDTD and extended to three dimensions.
- HSG provides an efficient Maxwell's Equations solver capable of fine geometry representation.
- Filtering was implemented to suppress late-time instabilities of HSG.
- Filter 33 shows the best instability suppression properties: $t_{inst,worst} = 5200$ time steps.
- Future work will focus on HSG parallelisation and further improvement of HSG stability.

Bibliography

[1] Jean-Pierre Bérenger. A FDTD Subgridding Based on Huygens Surfaces. *IEEE Antennas and Propagation International Symposium*, 2005, Washington D.C., USA.

[2] Jean-Pierre Bérenger. A Huygens Subgridding for the FDTD Method. *IEEE Antennas and Propagation*, 2005, November.

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